NASA Technical Memorandum 83737

Nonlinear Displacement Analysis of Advanced Propeller Structures Using NASTRAN

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August 1984

NASA

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ADVANCED PROPELLER STRUCTURES USING NASTRAN

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SUMMARY

The steady state displacements of a rotating advanced turboprop, SR3, were computed using the geometrically nonlinear capabilities of COSMIC NASTRAN Rigid Format 4 and MSC NASTRAN Solution 64. Displacements were computed for rotational speeds up to 10 000 rpm. The results showed that a complete non-linear analysis is required. Neither a linear elastic analysis nor a one step differential stiffness analysis were adequate. The inaccuracy of these linear analyses increased with increases in rotational speeds.

A modified Newton-Raphson algorithm used by MSC NASTRAN Solution 64 and an iterative scheme used by COSMIC NASTRAN in Rigid Format 4 were employed for predicting geometrically nonlinear response of SR3. Solution 64 was run two ways: (1) using a constant centrifugal body force computed from the undeformed blade, and (2) using a force field corrected in each iteration for changes in the blade's position. Rigid Format 4 does not possess this load updating capability. When the load updating feature of Solution 64 was not used, the two programs generated similar tip deflections and rotations. When the load updating capability of Solution 64 was used, the tip displacement varied significantly between MSC Solution 64 and COSMIC Rigid Format 4.

The SR3 turboprop blade has been shown to behave in a geometrically non-linear fashion under centrifugal loading. This nonlinear response becomes increasingly important at higher rotational speeds. Also, since MSC NASTRAN Solution 64 can correct the centrifugal load for changes in blade position, it is a more desirable analysis tool for predicting nonlinear behavior than COSMIC NASTRAN Rigid Format 4.

INTRODUCTION

Increasing concern with improving aircraft fuel efficiency has brought about renewed interest in propeller propulsion systems. Improved multi-bladed propellers, termed advanced turboprops, have the potential for reduced fuel consumption while maintaining the performance levels of modern turbofans. These turboprops feature thin, flexible, swept blades with complex structural properties. Extensive research in both analytical and experimental techniques has been and continues to be conducted by researchers in the field to better understand the structural and aerodynamic response of these complicated blades.

An important area to investigate is the computation of steady state deflections in rotating turboprop blades. Previous experience has demonstrated that linear analysis techniques are inaccurate for predicting the deflections

of these rotating blades. Because the blades are relatively flexible they respond to loading with relatively large deflections. This complicates the calculation of the blade's stiffness since the stiffness and deflections are mutually dependent. The computation of this behavior requires a geometrically nonlinear analysis.

The analysis is further complicated by the fact that the centrifugal loads are also displacement dependent. Since centrifugal force is proportional to the radius from the rotational axis the magnitude and distribution of centrifugal loads will change as the blade displaces. The research presented in this paper investigates this effect.

The purpose of the research presented in this paper is to investigate the use of NASTRAN, a well-known finite element program, for predicting steady state deflections of advanced turboprops subject to centrifugal loading. This computer program was selected due to its' nonlinear analysis capability and availability. Furthermore, it is universally accepted and has been used successfully to analyze geometrically nonlinear structural problems in the past. Two versions of NASTRAN were employed, the first was MSC (Macneal-Schwendler Corporation) NASTRAN Solution 64; the second was COSMIC (Computer Software Management and Information Center) NASTRAN Rigid Format 4. Both of these programs can be used for geometrically nonlinear analyses. It is convenient to use COSMIC NASTRAN Rigid Format 4 because the stiffness matrix generated in this program is in a format compatible for input into the subsequent aerodynamic flutter analysis. The turboprop blade was analyzed using MSC NASTRAN Solution 64 in order that the results from rigid format 4 could be compared to an independent nonlinear analysis.

A representative advanced turboprop, named SR3, was used as the model for investigating the use of NASTRAN for computing deflections of advanced turboprops. SR3 has most of the structurally related characteristics found in typical advanced turboprop blades (fig. 1).

BACKGROUND

At NASA Lewis Research Center COSMIC NASTRAN runs on a UNIVAC 1100 computer system which utilizes a 32 bit processor and uses double precision. MSC NASTRAN is run on a Cray 1-S. This program uses single precision and a 64 bit processor.

As previously mentioned, it is desirable to use COSMIC NASTRAN for the nonlinear analysis because the results from this program can easily be used in the aerodynamic flutter analysis. Since both the nonlinear analysis (Rigid Format 4) and the aerodynamic flutter analysis (Rigid Format 9) are run on the UNIVAC using COSMIC NASTRAN, it is relatively simple to transfer results from one program to the other. More specifically, it is necessary that the stiffness matrix generated from the nonlinear analysis in Rigid Format 4 be transferred to Rigid Format 9 for use in the flutter analysis. If the flutter analysis does not use a stiffness matrix that includes nonlinear effects, correct frequencies will not be computed and the results of the flutter analysis will be in error. The stiffness matrix from MSC Solution 64 cannot be used

for the flutter analysis because there is presently no practical method for transferring matrices between MSC and COSMIC NASTRAN. An effort is currently being made to eliminate this problem.

From among all the MSC and COSMIC NASTRAN solution sequences available for geometrically nonlinear response Solution 64 is thought to be the most complete. In order to verify the accuracy of Rigid Format 4, the results from this program are compared to the results from MSC NASTRAN Solution 64.

MSC NASTRAN Solution 64 for Geometrically Nonlinear Analysis

MSC NASTRAN Solution 64 provides a straightforward means for performing geometrically nonlinear analysis utilizing a modified Newton-Raphson algorithm (refs. 1 and 2). The algorithm used in this solution sequence is described in the MSC NASTRAN application manual and will be repeated here for the reader's convenience. To best understand the Newton-Raphson algorithm used by MSC NASTRAN, a simplified incremental scheme will first be described and then it will be shown how this scheme may be expanded into the Newton-Raphson algorithm.

The objective of a nonlinear analysis is to simulate the correct displacement versus load relationship. One means of accomplishing this is to use an incremental algorithm in which the nonlinear response is divided into a series of linear steps, each step representing an increase in load (fig. 2). Displacements are accumulated after each step and a new tangent stiffness is calculated based on the structure's deformed position. By using this approach the nonlinear response can be approximated by a series of linear segments. The load increment determines how close the approximation will match the actual response curve. The larger the load increment, the further the computed response will be from the actual response. When large load steps are used, the purely incremental algorithm will deviate considerably from the correct structural behavior. As shown in figure 2, while the actual displacement for applied load R is at D_a, the computed displacement is only at D_3. This deviation results in equilibrium conditions not being satisfied at the end of the load step. While the difference ΔR between externally applied loads and internal element forces should equal zero, they are actually equal to a nonzero value.

The error produced by the incremental approach can be eliminated in the limit by requiring that equilibrium conditions be satisfied at the end of each load step. This approach is analogous to the conventional Newton-Raphson algorithm. The MSC NASTRAN form of the Newton-Raphson approach requires that equilibrium be satisfied ($\Delta R = 0$) by balancing the externally applied loads and the internal forces at the end of each load step. This constraint prohibits the computed response from deviating from the actual response. This algorithm has the added advantage that, for typical structures where the load-displacement curve is relatively smooth, the entire load can be applied in the first step without effecting the accuracy of the final results.

The MSC NASTRAN form of the Newton-Raphson algorithm is based on an iterative solution of the equation of equilibrium. In matrix notation, this equation takes the form:

$$[K_i] \{\Delta D_{i+1}\} = \{\Delta R_i\} \tag{1}$$

Where the load imbalance $\{\Delta R_i\} = \{R_a\} - \Sigma[k_i] \{d_i\}$. $[K_i]$ and $[k_i]$ are the global and element stiffnesses respectively, $\{R_a\}$ is the applied load, $\{d_i\}$ are the element displacements, and $\{\Delta D_{i+1}\}$ is the increment in the global displacement at the end of the iteration. The global stiffness matrix $[K_i]$ includes both the elastic and differential stiffness matrices.

The Newton-Raphson algorithm is shown graphically in figure 3. For the initial iteration, the structure begins in its undeformed position with the internal forces set to zero ($\Sigma[k_i] \{d_i\} = \{0\}$. This reduces the right side of equation (1) to the applied load vector $\{R_a\}$.

By using the load $\{R_a\}$ and the tangent stiffness $[K_0]$ at the origin, the displacement $\{\Delta D_1\}$ is computed. After applying $\{\Delta D_1\}$ to the undeformed blade, the position of the displaced blade is established. Using this deformed shape, the internal forces ($\Sigma[k_i] \{d_i\}$) are generated along with a new tangent stiffness. The next iteration is initiated by computing a new set of displacements using the imbalance ($\{R_a\} - \Sigma[k_i] \{d_i\}$) as the applied load and the new tangent stiffness as the stiffness. This procedure is repeated until the load imbalance is reduced to within the desired tolerance. Reducing the tolerance produces more accurate results but requires additional iterations.

It is not mandatory for the load vector $\{R_a\}$ to be equal to the total external load in every iteration. Instead, $\{R_a\}$ can be only a fraction of the total external load in early iterations and then can be incremented to the full load in the final iterations. In the example shown in figure 3, $\{R_a\}$ is taken as the full externally applied load in every iteration. If the structure is highly nonlinear and the total external load is applied in the initial iterations the structure may move to an unstable condition. The advantage of using load increments is that the path to instability can be monitored.

In the modified Newton-Raphson approach the elastic stiffness matrix is not updated in each iteration. Instead, the original stiffness matrix derived from the undeformed geometry is used for every iteration. Using this original stiffness does not affect the final results because the final displacement is independent of the path used! (The element stiffnesses which are used to compute the load imbalance on the right side of eq. (1) are updated in each iteration.) By referring to figure 3, one can visualize the affect of using the same stiffness in every iteration. If $[K_0]$ is used as the stiffness in every iteration, the incremental displacements will be different from the displacements shown in figure 3 (altering the path), but the final converged displacement will still be equal to D_a.

The advantage of using an unaltered stiffness matrix is that the cost of forming and decomposing a new elastic stiffness matrix for each cycle is eliminated. The disadvantage is that the solution will require more iterations to converge. Although the elastic stiffness need not be updated with MSC NASTRAN, the user does have the option of computing a new differential stiffness matrix in any iteration. It is useful to be able to update the differential stiffnesses matrix since it is a function of element stresses and can change considerably with load increments.

MSC NASTRAN Solution 64 uses "subcases" to control the execution of the nonlinear analysis. Specifically, the number of subcases specified in the NASTRAN input data deck determines the number of iterations that will be performed. The user must specify at least two subcases. In the first subcase a linear elastic analysis is used to compute an initial deflected shape. This displaced shape is then used in the second subcase to compute the differential stiffness matrix along with a new set of displacements. Subsequent subcases are used for iterating on the equilibrium equation (eq. 1).

Solution 64 has two important features that increase the flexibility of the nonlinear analysis. First, Solution 64 has the ability to recompute the external loads before each iteration. This is particularly advantageous for centrifugal loads since they are dependent on the radius of the blade's mass from the rotational axis and will, therefore, change in each iteration as the blade deflects. The second asset of Solution 64 is its ability to apply a fraction of the full external load in earlier subcases. This advantage has previously been discussed. Neither of these capabilities are available in COSMIC NASTRAN Rigid Format 4.

COSMIC NASTRAN Rigid Format 4 for Differential Stiffness

Rigid Format 4 of COSMIC NASTRAN is designed to solve geometrically nonlinear problems (ref. 3). This rigid format uses an iterative solution sequence based on an extension of the one step differential algorithm. The governing equation for this rigid format is:

$$\left[[K] + [K_d (D_i)] \right] \{D_{i+1}\} = \{R_a\} \tag{2}$$

Where $[K]$ is the global elastic stiffness, $[K_d (D_i)]$ is the differential stiffness computed for the displaced blade at $\{D_i\}$, $\{R_a\}$ is the applied load, and $\{D_{i+1}\}$ is the displacement computed at the end of the iteration. In this equation the differential stiffness matrix $[K_d]$ is updated in each iteration whereas the elastic stiffness $[K]$ remains constant. Equation (2) is not guaranteed to converge to the correct displacement because this equation, unlike equation (1), contains no check to insure a small load imbalance between internal element forces and external body forces. Thus, for highly nonlinear structures, equation (2) may not converge.

Rigid Format 4 requires two subcases. The first subcase is used for performing linear elastic analyses without incorporating differential stiffness. The resulting element stresses from the first subcase are used in the second subcase to compute the differential stiffness matrix. Equation (2) is then repeated until the weighted difference between the differential stiffness in subsequent iterations is within the desired tolerance or until the maximum desired number of iterations has been completed. Both the tolerance and number of iterations can be controlled by the user. The linear elastic and initial differential analyses performed by Rigid Format 4 are comparable to the first two subcases of MSC NASTRAN Solution 64.

The major limitation of using Rigid Format 4 for computing deflections of rotating structures is the inability to compensate for the change in the centrifugal body force as the blade displaces. This limitation can introduce considerable error.

RESULTS

A comparison of results between MSC NASTRAN Solution 64 and COSMIC NASTRAN Rigid Format 4 was made using the advanced turboprop model SR-3. This turboprop blade is a small scale titanium model measuring 12-1/4 in. from the rotational axis to the blade tip. The blade thickness decreases from over one in. at the root to 0.016 in. at the tip. As shown in figure 1, the blade is both highly swept and twisted. This geometry is intended to optimize aerodynamic performance and minimize noise while maintaining structural integrity. The finite element model consists of 346 triangular plate elements and 206 grid points. The "CTRIA2" and "CTRIA3" elements were used for the COSMIC and MSC runs, respectively. The plate element formulations include both membrane and bending action. The base of the root of the blade is modeled as fully constrained.

MSC NASTRAN Solution 64 was used to compute steady state displacements of SR-3 at 3500, 7000, 8600, and 10 000 rpm. Two sets of displacement data were generated. The first set was generated using a constant centrifugal body force computed from the original, undeformed blade. The second set used a body force updated in each iteration based on the position of the deformed blade. Seven subcases (5 iterations) were used for Solution 64. Convergence was evaluated by comparing the difference in displacements between the sixth and seventh subcases which were on the order of only one tenth of one percent. The five iterations produced a ratio of unbalanced forces at unconstrained nodes to forces at the constrained root of 10^{-4}. This small ratio indicates that the load imbalance at the unconstrained nodes is relatively small and that equation (1) has adequately converged.

Steady state displacements were computed using COSMIC NASTRAN Rigid Format 4 at the same rotational speeds as MSC Solution 64. Only two iterations are required to produce converged displacements at 3500 rpm. Additional iterations changed the final displacements by only 1/2 percent. The number of iterations was increased to three by decreasing the default tolerance from 10^{-5} to 10^{-6} (PARAM, EPSIO, 1.E-6). At 10 000 rpm seven iterations were required for the same order of accuracy.

Similar finite element formulations are used for both COSMIC NASTRAN and MSC NASTRAN. As expected, the two programs computed similar, though not identical, displacements in the linear elastic subcase. The small discrepancy in the displacements is probably the result of either the minor difference in element formulations or the difference in criteria used by the two programs for testing small values of stiffness in the global stiffness matrix. Since the formulation for triangular plate elements does not include in-plane rotational stiffness, the assembled global stiffness matrix may contain very small or zero entries. These small values are usually removed by constraining appropriate rotational degrees of freedom. The default minimum value used by COSMIC NASTRAN is larger than the value used by MSC NASTRAN. Thus more constraints are present in the COSMIC runs. These additional constraints add stiffness to the blade and typically lower the Rigid Format 4 elastic displacements. However, the displacements in the subsequent nonlinear analysis do not appear to be affected by the additional constraints.

Tip deflection (defined as total deflection of midchord at blade tip) as a function of rotational speed is shown in figure 4. The general trend is for

deflections to increase up to around 7000 rpm and then level off. All three analyses follow this trend. COSMIC Rigid Format 4 and MSC Solution 64 produced similar results when centrifugal loads were not updated with changes in blade position. The difference in tip deflections between these two solutions with no load update decreased from 14 percent at 3500 rpm to only 5 percent at 10 000 rpm. There is a constant 0.01 inch difference in displacement between the two solutions above 3500 rpm. When the load updating capability of Solution 64 was utilized, the deflections were considerably different. The deflection curve generated using the load updating algorithm shows larger deflections than the other two curves (32 percent larger at 10 000 rpm). This was expected because the centrifugal force actually increases as the blade straightens out and the radius of the blade's mass from the rotational axis increases. It is evident that at high rotational speeds a load updating algorithm must be used to account for the large changes in blade position.

Figure 5 shows blade tip rotation as a function of rotational speed. Tip rotation is defined as blade tip chord twist about the pitch axis in a plane normal to the pitch axis. The pitch axis is shown in figure 1. The general trend in this figure is for the blade to untwist as rotational speed increases. As with tip deflections, Solution 64 with load updating produced the largest rotations. The differences between the load updating and constant load solutions were not as notable for tip rotations as they were for tip midchord deflections. The variation in tip rotations between Rigid Format 4 and Solution 64 with load updating was 19 percent at 10 000 rpm.

Figure 6 shows a comparison between the three levels of analysis used in MSC NASTRAN Solution 64. As expected, the linear elastic analysis (subcase 1) over predicted the tip deflection. This is due to the stiffening effect from the centrifugal force field which was not included in the linear analysis. The relationship between deflection and rotational speed is parabolic since centrifugal force is proportional to the square of rotational speed. The deflection computed in the linear analysis is in contradiction to the trend predicted by the nonlinear analysis shown in the same figure. The nonlinear analysis shows the tip deflection leveling off. The displacements from subcase 1 are considerably greater than those computed in subcase 2, which includes centrifugal stiffening effects. While the tip displacement was over 0.80 in. at 10 000 for the linear solution, it was only 0.21 in. for the linear elastic plus differential analysis. The difference in deflections between the linear and nonlinear solutions is shown by comparing subcases 1 and 2 to subcase 7. The difference in tip deflection at 10 000 rpm, between subcase 7 and subcase 1 and 2 is 75 and 33 percent, respectively. This large variation between the nonlinear and linear subcases is a good indicator of the blade's strong nonlinear behavior.
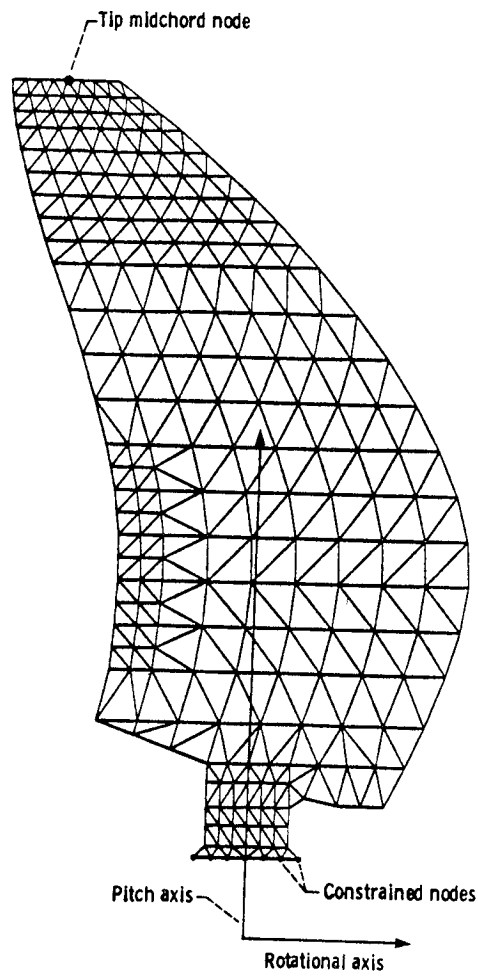

CONCLUDING REMARKS

Because of its ability to update the displacement dependent centrifugal force during the solution process, MSC NASTRAN Solution 64 was shown to be superior to COSMIC NASTRAN Rigid Format 4 for the geometrically nonlinear analysis of advanced propeller blades. However, it cannot be concluded that

MSC NASTRAN Solution 64 accurately predicts the steady displacement of advanced turboprop blades until additional analytical and experimental studies are completed.

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Tip midchord node

Pitch axis

Constrained nodes

Rotational axis

(a) SR3 turboprop finite element model.

Figure 1.

(b) SR3 turboprop finite element model.

Figure 1. - Concluded.

Load

Purely incremental

R

ΔR

Actual response curve

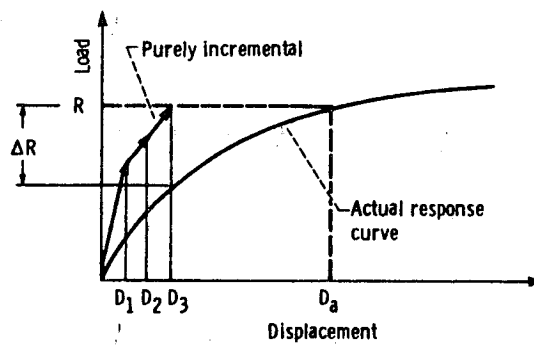D_1 D_2 D_3

D_a

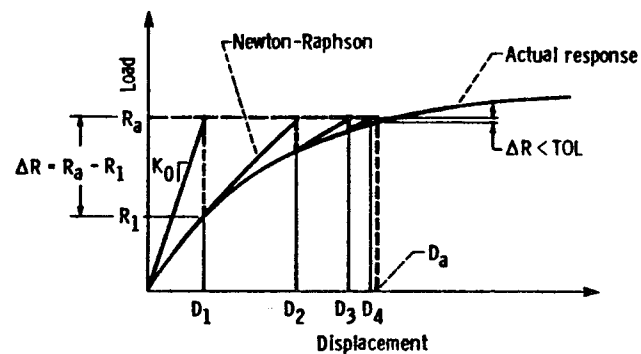Displacement

Figure 2. - Incremental algorithm for nonlinear analysis.

Figure 3. - Newton-Raphson solution for nonlinear analysis.
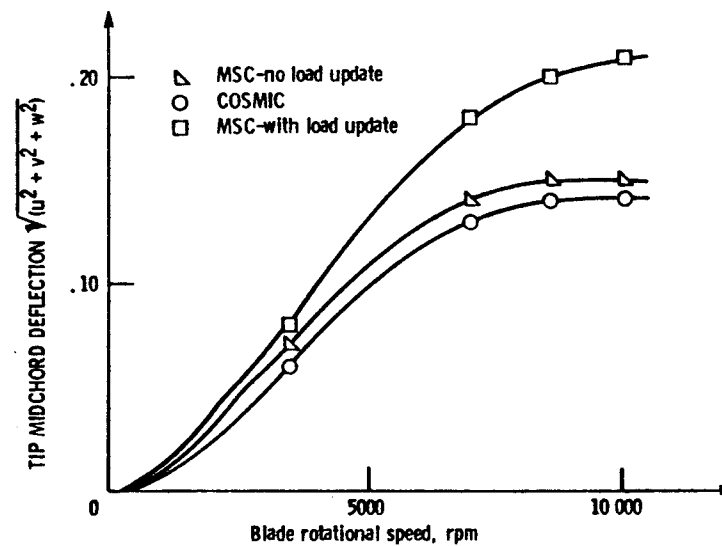

Figure 4. - SR3 tip displacement versus rotational speed.
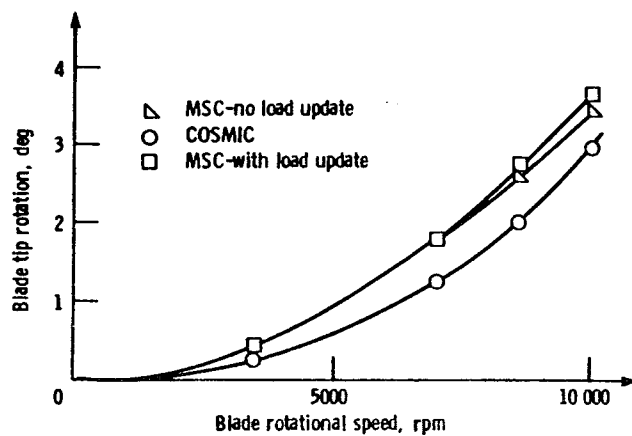

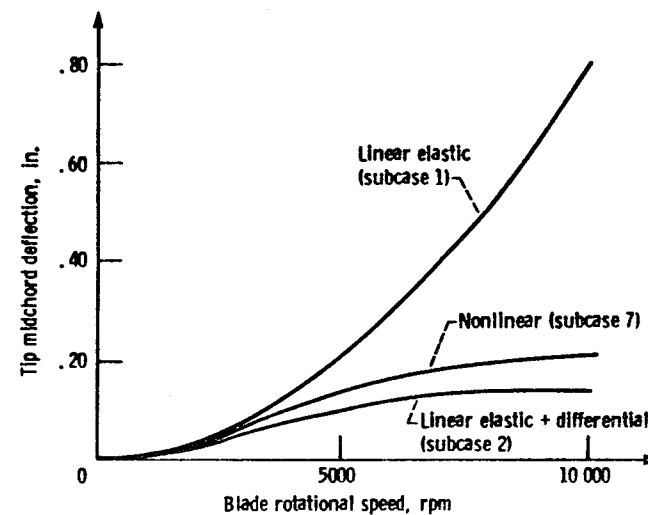Figure 5. - SR3 tip rotation versus rotational speed.

Figure 6. - Comparison of linear elastic, differential, and nonlinear analyses used by MSC solution 64.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-83737		

4. Title and Subtitle	5. Report Date
Nonlinear Displacement Analysis of Advanced Propeller Structures Using NASTRAN	August 1984
	6. Performing Organization Code
	535-03-12

7. Author(s)	8. Performing Organization Report No.
Charles Lawrence and Robert E. Kielb	E-2222
	10. Work Unit No.

9. Performing Organization Name and Address	11. Contract or Grant No.
National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	
	13. Type of Report and Period Covered

12. Sponsoring Agency Name and Address	Technical Memorandum
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The steady state displacements of a rotating advanced turboprop are computed using the geometrically nonlinear capabilities of COSMIC NASTRAN Rigid Format 4 and MSC NASTRAN Solution 64. A description of the modified Newton-Raphson algorithm used by Solution 64 and the iterative scheme used by Rigid Format 4 is provided. A representative advanced turboprop, SR3, was used for the study. Displacements for SR3 are computed for rotational speeds up to 10 000 rpm. The results show Solution 64 to be superior for computing displacements of flexible rotating structures. This is attributed to its ability to update the displacement dependent centrifugal force during the solution process.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Propellers NASTRAN Nonlinear	Unclassified - unlimited STAR Category 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		